Solving Polynomial Equations

In today’s lesson you will:
• factor polynomials.
• solve polynomial equations by factoring.
Sum and Difference of Cubes

Complete the table to summarize the factoring techniques for cubic polynomials.

<table>
<thead>
<tr>
<th>Factoring Technique</th>
<th>General Case</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sum of Two Cubes</td>
<td>$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$</td>
</tr>
<tr>
<td>Differences of Two Cubes</td>
<td>$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$</td>
</tr>
</tbody>
</table>

What is a polynomial called that cannot be factored?

Prime
Sum and Difference of Cubes

Factor each polynomial. If the polynomial cannot be factored, write prime.

\[ x^3 - 400 \]  
\[ 8x^3 + 64 \]  
\[ v^3w + 125w^4 \]  
\[ w(v^3 + 125w^3) \]  
\[ w(v + 5w)(v^2 - 5vw + 25w^2) \]

\[ \text{not a perfect cube} \]  
\[ \text{Doesn't factor} \]  
\[ \text{Prime} \]  
\[ (x^3 + 8) \]  
\[ (x+2)(x^2 - 2x + 4) \]  
\[ (v^3 + (5w)^3) \]  
\[ a^3 + b^3 \]
<table>
<thead>
<tr>
<th>Number of Terms</th>
<th>Factoring Technique</th>
<th>General Case</th>
</tr>
</thead>
<tbody>
<tr>
<td>Any number</td>
<td>Greatest Common Factor</td>
<td>$ax + ay = a(x + y)$</td>
</tr>
<tr>
<td>Two</td>
<td>Difference of Two Square</td>
<td>$a^2 - b^2 = (a + b)(a - b)$</td>
</tr>
<tr>
<td></td>
<td>Difference of Two Cubes</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Sum of Two Cubes</td>
<td>$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$</td>
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<td></td>
<td>Sum of Two Cubes</td>
<td>$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$</td>
</tr>
<tr>
<td>Three</td>
<td>General Trinomials</td>
<td>$x^2 + (a + b)x + ab = (x + a)(x + b)$</td>
</tr>
<tr>
<td>Four</td>
<td>Grouping</td>
<td>$ax + ay + bx + by = a(x + y) + b(x + y)$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$= (a + b)(x + y)$</td>
</tr>
</tbody>
</table>
Factor $x^3 + 2x^2 + 8x + 16$

$x^2(x + 2) + 8(x + 2)$

$\begin{array}{c}
\times \\
\times
\end{array}$

$(x^2 + 8)(x + 2)$

Grouping

$\frac{x^3 - 2x^2 + 4x - 8}{x^2(x - 2) + 4(x - 2)}$

$\begin{array}{c}
\times \\
\times
\end{array}$

$(x^2 + 4)(x - 2)$
Factoring By Grouping

Factor each completely.

1) \(8r^3 - 64r^2 + r - 8\)
   \[
   \frac{8r^2(r - 8) + 1(r - 8)}{(8r^2 + 1)(r - 8)}
   \]

2) \(12p^3 - 21p^2 + 28p - 49\)
   \[
   3p^2(4p - 7) + 7(4p - 7)
   \]
   \[
   (3p^2 + 7)(4p - 7)
   \]

3) \(12x^3 + 2x^2 - 30x - 5\)
   \[
   \frac{2x^2(6x + 1) - 5(6x + 1)}{(2x^2 - 5)(6x + 1)}
   \]

4) \(6v^3 - 16v^2 + 21v - 56\)
   \[
   2v^2(3v - 8) + 7(3v - 8)
   \]
   \[
   (2v^2 + 7)(3v - 8)
   \]

5) \(63n^3 + 54n^2 - 105n - 90\)

6) \(21k^3 - 84k^2 + 15k - 60\)
Quadratic Form

Words
An expression that is in quadratic form can be written as $au^2 + bu + c$ for any numbers $a$, $b$, and $c$, $a \neq 0$, where $u$ is some expression in $x$. The expression $au^2 + bu + c$ is called the quadratic form of the original expression.

Example
$x^6 + 5x^3 - 24 = (x^3 + 8)(x^3 - 3)$
Factor, if possible.

a. \( x^4 + 2x^2 + 1 \)
   \[ (x^2 + 1)(x^2 + 1) \]

b. \( x^4 - 2x^3 - 1 \)
   Prime